

WH MATH SUMMER PACKET

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|-------------------|---|
| For whom: | All students taking math next year at WH (like... all of you 😊) |
| Goals: | <ul style="list-style-type: none"> To review important material from previous courses which you are expected to be fluent. To identify key concepts and skills you need to know for the next level. |
| Resources: | <p>Feel free to use whatever resources you need to review this material.</p> <p>You can still access your MathXL account that you opened last year. MathXL has an extensive library of sample problems and practice questions</p> |
| Reminder: | Remember, there are often different ways to solve a problem. You may use a different process or strategy than someone helping you and that doesn't mean you are wrong. |

Answers will be posted.

For students going into 2A: Complete parts 1-5

For students going into 2B: Complete parts 1-5, 6-10, part of Part 11, and 18 if you have had geometry.

For students going into Precalculus: Complete all parts: 1-5, 6-10, 11, and 12-18

For students going into Calculus 1: Complete all parts: 1-5, 6-10, 11, and 12-18 (trigonometric function review is not included as it is not needed for Calc 1)

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Part 1: Exponent Rules

Defining a^n :

If n is a positive integer, then a^n (read as " a to the n ") is the product of n factors of a .

$$a^n = \underbrace{a \cdot a \cdot a \cdots a}_{n \text{ factors of } a}$$

Rules of Exponents:

$a^m \cdot a^n = a^{m+n}$ or "when you multiply like bases you add the exponents."

Example: $x^3 \cdot x^5 = x^8$

$(a^m)^n = a^{mn}$ or "when you raise a power to a power you multiply the exponents."

Example: $(x^4)^3 = x^{12}$

$(ab)^n = a^n b^n$ or "when you raise a product to a power you raise each factor to the power."

Example: $(xy)^3 = x^3 y^3$

$\frac{a^m}{a^n} = a^{m-n}$ or "when you divide like bases you subtract the exponents."

Example: $\frac{x^5}{x^7} = x^{5-7} = x^{-2}$

$a^{-m} = \frac{1}{a^m}$ or "when there is a negative exponent in the numerator, it moves from the numerator to the denominator and the exponent value changes sign."

Example: $x^{-3} = \frac{1}{x^3}$ or

$\frac{1}{a^{-r}} = a^r$ or "when there is a negative exponent in the denominator, it moves from the denominator to the numerator and the exponent value changes sign."

Example: $\frac{1}{x^{-5}} = x^5$

$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ or "when you raise a quotient to a power, raise both the numerator and the denominator to the power."

Example: $\left(\frac{x}{y}\right)^3 = \frac{x^3}{y^3}$

$a^0 = 1$ or "anything to the power of 0 is 1."

| | |
|--|---|
| Examples: Simplify: $2x^5(3x^7)$ $= 2 \cdot 3 \cdot x^5 \cdot x^7$ $= 6x^{5+7}$ $= 6x^{12}$ | Simplify: $4x^3(x^2y)^7(y^2)^0$ $= 4x^3x^{2 \cdot 7}y^7 \cdot 1$ $= 4x^3x^{14}y^7$ $= 4x^{17}y^7$ |
| Simplify: $\frac{5x^9}{10x^2}$ $= \frac{5}{10} \cdot x^{9-2}$ $= \frac{1}{2}x^7 \text{ or } \frac{x^7}{2}$ | Simplify: $\frac{14x^{12}y^2}{10x^2y}$ $= \frac{14}{10} \cdot \frac{x^{12}}{x^2} \cdot \frac{y^2}{y}$ $= \frac{7}{5} \cdot x^{12-2} \cdot y^{2-1}$ $= \frac{7}{5}x^{10}y \text{ or } \frac{7x^{10}y}{5}$ |

FOR YOU TO DO: Follow the instructions provided in problem 1. In each of problems 2-11, use the rules of exponents to simplify the given expression.

| | |
|--|--------------------------|
| 1. Is each expression below is equal to 6^5 (say "yes" or "no"). If it is, show why it is. a. $6^2 + 6^3$ b. $6^2 \cdot 6^3$ c. $\frac{6^{20}}{6^4}$ d. $(6)(6^4)$ e. $(6^2)^3$ | |
| 2. $x^7 \cdot x^3$ | 3. $(2^3)^5 \cdot (2^6)$ |

| | |
|--|--|
| 4. $(-9h)^2$ | 5. $n^5k^{10}n^6$ |
| 6. $(4g^3)^2 \cdot (-3g)$ | 7. $(a^{11} \cdot b \cdot c \cdot d^{31})^0$ |
| 8. $\frac{x^{12}}{3x^8}$ | 9. $\left(\frac{5n}{7m}\right)^2$ |
| 10. $\frac{3u^5w^4}{2uw^7}$ | 11. $g^2h^5(gh^4)^{10}$ |
| 12. $\frac{3x^{-2}y^{-4}}{2w^5z^{-2}}$ | 13. $\frac{x^{-2}y^4z^4w^{-3}}{4x^3y^2z^{-1}w^{-5}}$ |

Part 2: Multiplication and Division with Fractions

For this section we are focusing on working with **cross-cancellation**.

What is cross-cancellation? Cross-cancellation is a shortcut that you can use to make multiplying fractions easier, and eliminate simplifying at the end.

You can always convert a fraction division problem to a fraction multiplication problem. Just take the reciprocal of the divisor (flip the bottom or denominator).

Example

We want to perform the following division without a calculator: $\frac{30}{63} \div \frac{88}{24}$

Step 1: Flip and Multiply

$$\frac{30}{63} \div \frac{88}{24} = \frac{30}{88} \times \frac{24}{63}$$

Step 2: Prime Factor all numbers

$$\frac{30}{88} \times \frac{24}{63} = \frac{2 \times 3 \times 5}{2 \times 2 \times 2 \times 11} \times \frac{2 \times 2 \times 2 \times 3}{3 \times 3 \times 7}$$

Step 3: Cancel/divide out common factors in each fraction individually

$$\frac{\cancel{2} \times 3 \times 5}{\cancel{2} \times 2 \times 2 \times 11} \times \frac{2 \times 2 \times 2 \times 3}{3 \times 3 \times 7}$$

Step 4: "Cross-cancel"; that is cancel/divide out common factors across fractions

$$\frac{\cancel{3} \times 5}{\cancel{2} \times 2 \times 11} \times \frac{\cancel{2} \times \cancel{2} \times 2}{\cancel{3} \times 7}$$

Step 5: Multiply the numerators together, and multiply the denominators together

$$\frac{5}{11} \times \frac{2}{7} = \frac{10}{77}$$

You can save work if the numbers are relatively small and you can see some common factors. In such a case you don't have to find the complete prime factorization. Instead, we usually cross out numbers across a diagonal and write replacement numbers next to them with the Greatest Common Factors removed.

Example

Calculate the product: $\frac{16}{15} \times \frac{21}{24}$

Step 1: Identify the GCF of the numbers across from each other

$$\frac{16}{15} \times \frac{21}{24}$$

The GCF of 16 and 24 is 8, the GCF of 15 and 21 is 3

Step 2: "Cross-cancel"

$$\begin{array}{c} 2 \quad \cancel{16} \quad 7 \\ \quad \times \quad \cancel{21} \quad 3 \\ \hline \cancel{15} \quad \cancel{24} \end{array}$$

Step 3: Multiply the numerators together, and multiply the denominators together

$$\frac{2}{5} \times \frac{7}{3} = \frac{14}{15}$$

FOR YOU TO DO: Perform the multiplication or division.

1. $\frac{14}{9} \times \frac{7}{10}$

2. $\frac{10}{11} \times \frac{22}{25}$

3. $\frac{5/6}{3/2}$

4. $\frac{7/9}{14/3}$

5. $\frac{4/15}{20/9}$

6. $\frac{12/35}{3/55}$

7. $\frac{56/99}{21/44}$

8. $\frac{18/32}{24/80}$

Part 3: Finding the GCF

Defining the GCF:

The GCF, or Greatest Common Factor, among a set of numbers is the largest factor all the numbers have in common.

e.g., the GCF of 10, 15, and 35 is 5

e.g. the GCF of x^4 , x^5 , and x^8 is x^4

Examples:

What is the GCF of $10n^4$ and $-16n^6$?

Answer: $2n^4$

Evidence: $10n^4 = 2 \cdot 5 \cdot n \cdot n \cdot n \cdot n$

$-16n^6 = -1 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot n \cdot n \cdot n \cdot n \cdot n \cdot n$

So the greatest common factor includes 2 and four n 's

What is the GCF of $(x-2)^2$ and $(x-2)^5$?

Answer: $(x-2)^2$

Evidence: $(x-2)^2 = (x-2)(x-2)$ and $(x-2)^5 = (x-2)(x-2)(x-2)(x-2)(x-2)$ so they have two factors of $(x-2)$ in common

FOR YOU TO DO:

Determine the GCF for each group of expressions.

| | |
|------------------------|-----------------------------------|
| 1. $9x^2$ and $15x^3$ | 2. p^3q^4 and p^3q^5 |
| 3. $9x^3$ and $36x$ | 4. p^2q^4 and p^7q |
| 5. x^2 and y^2 | 6. ab , ac , and bc |
| 7. ab and ac | 8. p^2q^5 , p^3q^4 and p^7q |
| 9. $15a^2$ and $21b^2$ | 10. $9(x+3)^3$ and $6(x+3)^2$ |

Part 4 : Finding the LCM (least common multiple) or in fractions LCD (lowest common denominator)

Defining the LCM:

The LCM, or Least Common Multiple is the smallest number that is a multiple of both or all of the numbers.

e.g., the LCM of 5 and 3 is 15

e.g. the LCM of x and 5 is $5x$

e.g. the LCM of $3x^2$ and $5x$ is $15x^2$

To find least common multiple use prime factorization:

LCM of 36 and 48

Prime factors of 36: $2 \cdot 2 \cdot 3 \cdot 3$

Prime factors of 48: $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$

Least common multiple is the most prime factors that are in either.

So four "2s" and two "3s" = $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 = 144$

You need a common denominator (bottom) to add or subtract fractions. By find the lowest common denominator (LCD), the answer will not need to be reduced!

LCD is the same as LCM, $\frac{1}{36} + \frac{1}{48} = \frac{1}{36} \left(\frac{4}{4}\right) + \frac{1}{48} \left(\frac{3}{3}\right) = \frac{4}{144} + \frac{3}{144} = \frac{7}{144}$

Examples:

What is the LCM of 15 and 6?

Answer: 30

Evidence: $15 = 3 \cdot 5$

$6 = 2 \cdot 3$

So the least common multiple is: $2 \cdot 3 \cdot 5 = 30$

What is the LCM of 35 and 25?

Answer: 175

Evidence: $35 = 5 \cdot 7$

$25 = 5 \cdot 5$

So the least common multiple is: $5 \cdot 5 \cdot 7 = 175$

Add these fractions:

$$\frac{3}{25} + \frac{2}{15} = ?$$

Answer: $\frac{19}{75}$

Evidence: $\frac{3}{5 \cdot 5} \left(\frac{3}{3}\right) + \frac{2}{5 \cdot 3} \left(\frac{5}{5}\right) = \frac{9}{75} + \frac{10}{75} = \frac{19}{75}$

Subtract these fractions:

$$\frac{1}{4x^2y} - \frac{3}{8xy^3}$$

Answer: $\frac{2y^2-3x}{8x^2y^3}$

Evidence: $\frac{1}{4 \cdot x \cdot x \cdot y} \left(\frac{2 \cdot y \cdot y}{2 \cdot y \cdot y}\right) - \frac{3}{8xy^3} \left(\frac{x}{x}\right) = \left(\frac{2y^2-3x}{8x^2y^3}\right)$

FOR YOU TO DO:

Determine the LCM for each group of expressions

| | |
|-------------------|-----------------------|
| 1. $24, 32$ | 2. $18xy^2, 15y^3$ |
| 3. $20x^3, 40x^4$ | 4. $24x^2yz, 28xyz^2$ |

FOR YOU TO DO:

Add or subtract the fractions.

| | |
|------------------------------------|--------------------------------------|
| 5. $\frac{1}{2} + \frac{5}{7}$ | 6. $\frac{1}{12} - \frac{5}{42}$ |
| 7. $\frac{5}{3x^2} - \frac{7}{2x}$ | 8. $\frac{y}{14xz^2} + \frac{x}{6z}$ |

Part 5: Linear Function and domain, range and solving.

Definition: The **domain** is all possible values of the independent variable or the input (usually x).

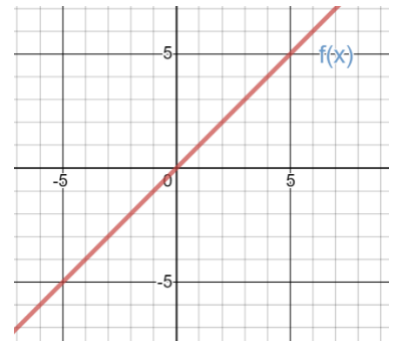
Definition: The **range** is all set of possible dependent variable or output values (usually y).

Notation: Consider $y = f(x) = 3x + 2$.

- The independent variable is x . The domain is all real numbers, written \mathbb{R} , and we can write $x \in \mathbb{R}, x = \mathbb{R}, x \in (-\infty, \infty)$ or $x = (-\infty, \infty)$
- The dependent variable is y . The range is y equals all real numbers or we can write $y \in \mathbb{R}$ or $y = (-\infty, \infty)$.
- The symbol " \in " means "can equal" or "is equal to".

$$f(x) = x$$

- Linear function.
- Domain $x \in \mathbb{R}$
- Range $y = f(x) \in \mathbb{R}$



Note: Three form of linear equations we use are:

Slope intercept form $y = mx + b$

- m = slope
- $(0, b)$ is the y -intercept
- (x, y) are any point on the line

Point slope form $(y - y_0) = m(x - x_0)$

- m = slope
- (x_0, y_0) is a known point on the line
- (x, y) are any point on the line

Standard form $Ax + By = C$ where A, B and C are integers (not fractions).

- x -intercept (where $y = 0$) is $\left(\frac{C}{A}, 0\right)$
- y -intercept (where $x = 0$) is $\left(0, \frac{C}{B}\right)$

The slope of a line can be found using any two points (x_1, y_1) and (x_2, y_2)

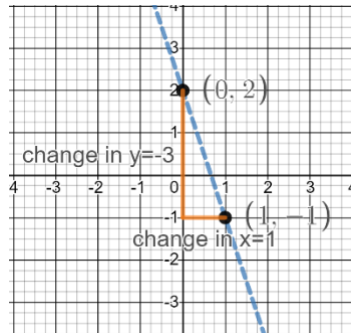
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 1: Graph $f(x) = -3x + 2$. State the domain and range.

- 1) Graph y -int $(0,2)$
- 2) Graph another point using

$$\text{slope} = -3 = \frac{-3}{1}$$

$$\frac{\text{"rise"}}{\text{"run"}} = \frac{\Delta y}{\Delta x}$$



Domain: $x \in \mathbb{R}$

Range: $y \in \mathbb{R}$

Example 3: Write the equation of a line with $m = \frac{-1}{2}$ and including $(1, -2)$ in slope intercept form.

- 1) Start with point slope form (we don't have the y -int)

$$(y - (-2)) = -\frac{1}{2}(x - 1)$$

- 2) Simplify

$$(y + 2) = -\frac{1}{2}x + \frac{1}{2}$$

$$y = -\frac{1}{2}x + \frac{1}{2} - 2$$

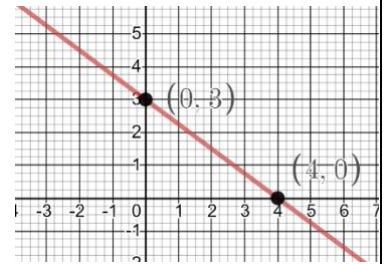
$$y = -\frac{1}{2}x - \frac{3}{2}$$

Example 2: Graph $3x + 4y = 12$. State the domain and range.

- 1) Graph x -int $(4,0)$
- 2) Graph y -int $(0,3)$
- 3) Connect the points.

Domain: $x \in \mathbb{R}$

Range: $y \in \mathbb{R}$



You could rewrite the equation in slope intercept form, but this is another strategy.

Example 4: Write the equation of a line in slope intercept form that include $(1,3)$ and $(-2,9)$

- 1) Find the slope

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 3}{(-2) - (1)} = \frac{6}{-3} = -2$$

- 2) use point slope form (we don't have the y -int) with either point.

$$(y - 3) = -2(x - 1)$$

- 3) Simplify

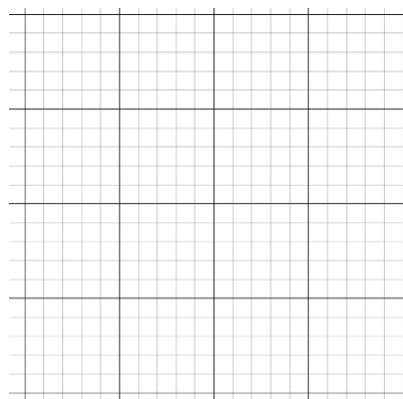
$$(y - 3) = -2x + 2$$

$$y = -2x + 2 + 3$$

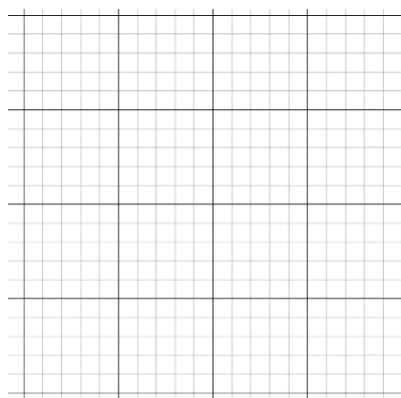
$$y = -2x + 5$$

FOR YOU TO DO: Graph the following functions and find the domain and range.

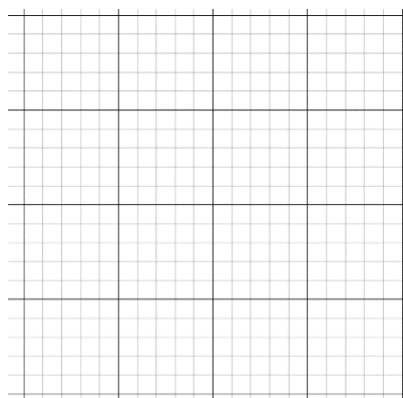
1. Graph $f(x) = -4x - 5$. State the domain and range.



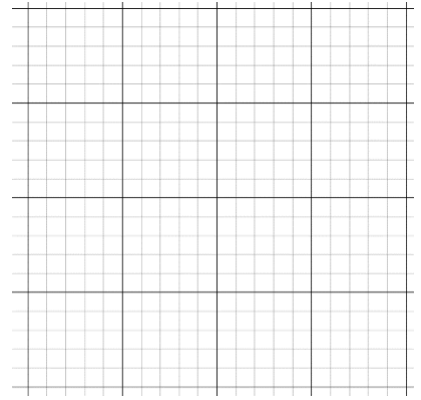
2. Graph $f(x) = 2x + 4$. State the domain and range.



3. Graph $-3x + y - 5 = 0$. State the domain and range.



4. Graph $2x - 6y = 18$. State the domain and range.



FOR YOU TO DO: Find the equations for the following lines.

1. Find the equation of a line with $m = 3$ and including point $(-1, 7)$ in slope intercept form.

2. Find the equation of a line through point $(-4, 2)$ and $m = \frac{-1}{4}$ in slope intercept form.

3. Write an equation with x -intercept = 3 and y -intercept = 6 in slope intercept and standard form.

4. Write an equation of a line through $(-2, 5)$ and $(1, -1)$ in slope intercept and standard form.

Part 6 : Transformations

When we have a function $f(x)$ we can “transform” it and create a new function using the general transformation formula

$$g(x) = \pm a f(x - h) + k.$$

We say that $g(x)$ is a transformation of $f(x)$.

The order of operations matters.

1. Shift horizontally by h units.
2. Stretch vertically by a (if a is a fraction, we compress the graph)
3. \pm If there is a negative sign we reflect the function over the x -axis.
4. Shift vertically by k units.

Example 1:

Describe the order of transformations to obtain $h(x)$ from $f(x)$.

a. $h(x) = 2f(x - 3) + 4$

1. Shift horizontally right by 3 units
2. Stretch vertically by a factor of 2
3. Shift vertically up by 4 units.

b. $h(x) = -f(x + 4) + 2$

Rewrite $h(x) = -f(x - (-4)) + 2$

1. Shift horizontally left by 4 units
2. Reflect over the x-axis
3. Shift vertically up by 2 units

c. $h(x) = \frac{2}{3}f(x - 3) - 4$

1. Shift horizontally right by 3 units
2. Compress vertically by a factor of $\frac{2}{3}$
3. Shift vertically down by 4 units..

d. $h(x) = -\frac{1}{2}f(x + 4) - 7$

1. Shift horizontally left by 4 units
2. Compress vertically by a factor of $\frac{1}{2}$
3. Reflect over the x-axis.
4. Shift vertically down by 7 units.

Example 2: Given the parent function $f(x) = x^2$ graph $g(x) = -2f(x+1) + 3$

Step 1: rewrite the equation: $g(x) = -2f(x - (-1)) + 3$

Step 2: Create a table of values for the parent function and apply the transformations.

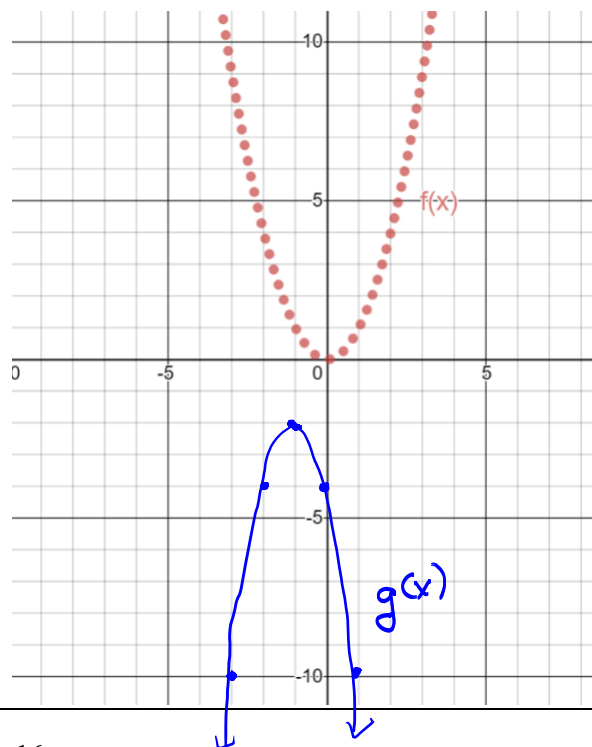
- Pick enough points for the parent function so that you can sketch the graph of the transformation accurately.
- Subtract 1 from each x-coordinate to shift the function to the left.
- For each y-coordinate
 Multiply by 2 to stretch it vertically
 Multiply by -1 to reflect it over the x-axis
 Add 3 to shift it vertically up by 3

| x transformations | Parent function points | | y transformations | | |
|----------------------|---------------------------|--------|-------------------|--------------|----|
| -1 | x | y=f(x) | $\cdot 2$ | $\cdot (-1)$ | +3 |
| -3 | -2 | 4 | 8 | -8 | -5 |
| -2 | -1 | 1 | 2 | -2 | 1 |
| -1 | 0 | 0 | 0 | 0 | 3 |
| 0 | 1 | 1 | 2 | -2 | 1 |
| 1 | 2 | 4 | 8 | -8 | -5 |

Step 3: Graph the transformed points

The transformed coordinates are:

(-3, -5)
 (-2, 1)
 (-1, 3)
 (0, 1)
 (1, -5)



Example 3: Given the graph of the following function, graph the transformation

$$h(x) = -\frac{1}{2}f(x+2) - 3 \text{ equation in general form.}$$

Step 1: Rewrite the equation as $h(x) = -\frac{1}{2}f(x - (-2)) - 3$

Step 2: Set up a table of key values using points from the graph and apply the transformations.

Pick points that will help you graph the transformation:

$(-6,3)$

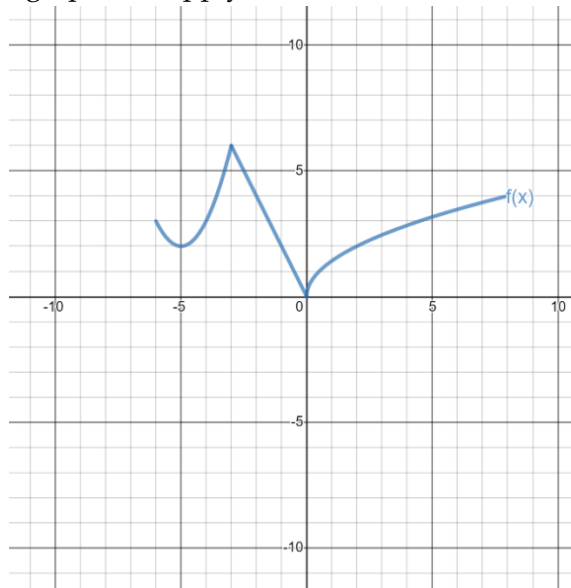
$(-5,2)$

$(-3,6)$

$(0,0)$

$(2,2)$ to help with the shape

$(8,4)$



| x transformations | Parent function points | | y transformations | | |
|-------------------|------------------------|--------|---------------------|----------------|-----------------------------------|
| -2 | x | y=f(x) | $\cdot \frac{1}{2}$ | $\cdot (-1)$ | -3 |
| -8 | -6 | 3 | $\frac{3}{2}$ | $-\frac{3}{2}$ | $-\frac{9}{2}$ or $-4\frac{1}{2}$ |
| -7 | -5 | 2 | 1 | -1 | -4 |
| -5 | -3 | 6 | 3 | -3 | -6 |
| -2 | 0 | 0 | 0 | 0 | -3 |
| 0 | 2 | 2 | 1 | -1 | -4 |
| 6 | 8 | 4 | 2 | -2 | -5 |

Step 3: Graph the transformed points.

$$\left(-8, -\frac{9}{2}\right)$$

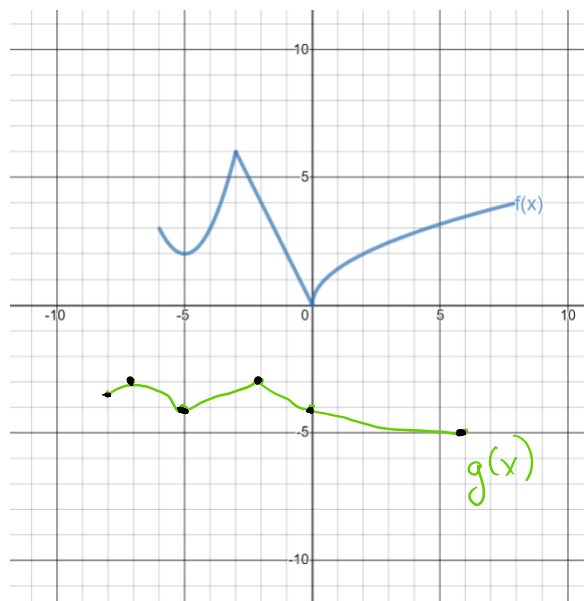
$$(-7, -4)$$

$$(-5, -6)$$

$$(-2, -3)$$

$$(0, -4)$$

$$(6, -5)$$



FOR YOU TO DO:

Describe the order of transformations to obtain $g(x)$ from $f(x)$.

1. $g(x) = \frac{3}{5}f(x-2) + 4$

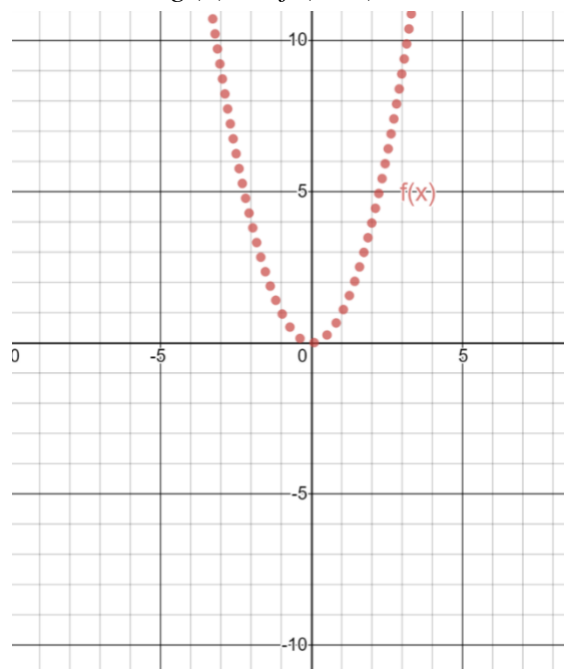
2. $g(x) = -2f\left(x + \frac{1}{2}\right) + 4$

3. $g(x) = -4f(x+3) - 2$

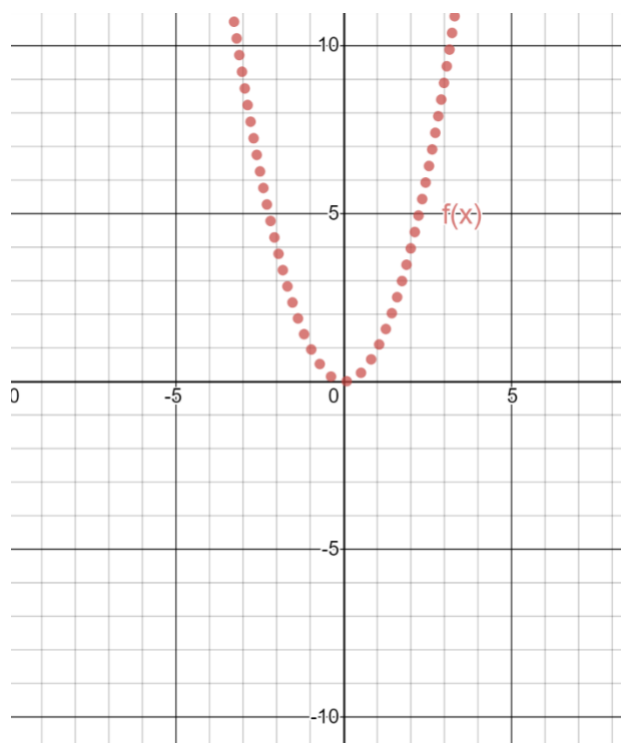
4. $g(x) = -\frac{3}{2}f\left(x - \frac{3}{5}\right) + \frac{3}{4}$

FOR YOU TO DO:

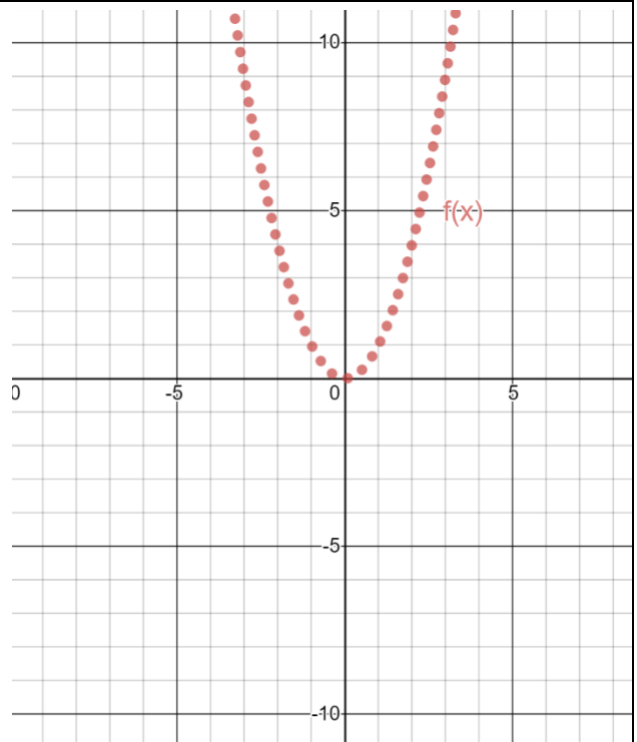
1. For the parent function $f(x) = x^2$, graph the transformation $g(x) = 2f(x-1) + 3$



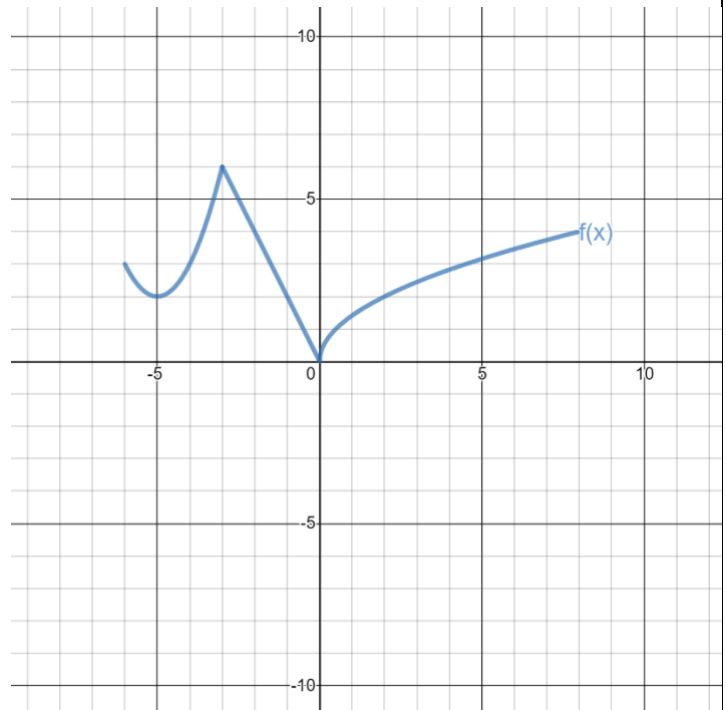
2. For the parent function $f(x) = x^2$, graph the transformation $g(x) = -f(x-1) - 2$



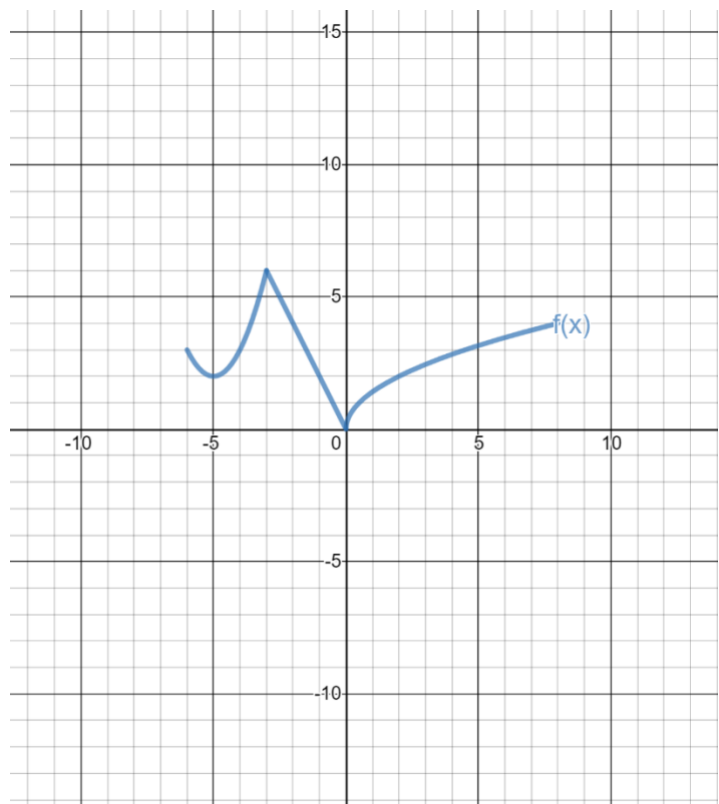
3. For the parent function $f(x) = x^2$, graph the transformation $g(x) = -f(x+2) - 3$



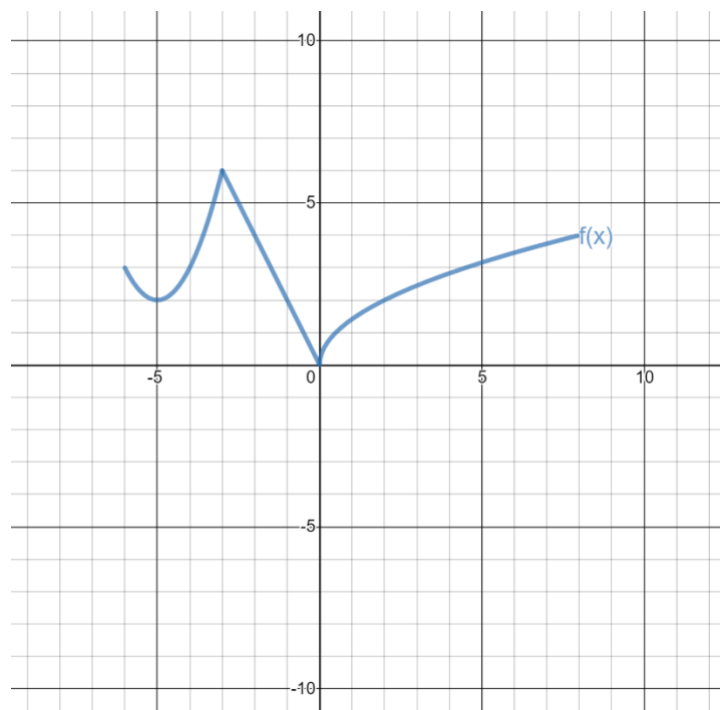
4. Given the graph of $f(x)$ below, graph the transformation $g(x) = f(x-1) + 2$



5. Given the graph of $f(x)$ below, graph the transformation $g(x) = -2f(x+3) + 1$



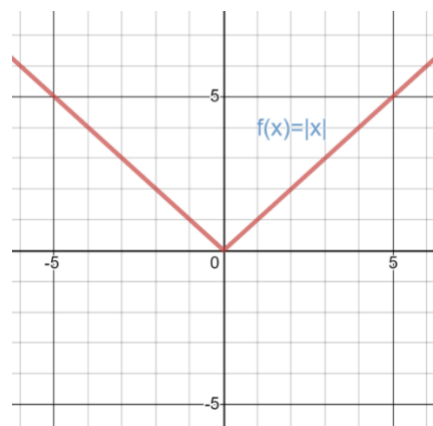
6. Given the graph of $f(x)$ below, graph the transformation $g(x) = -f(x-3) - 4$



Part 7: Absolute Value Functions and domain, range and solving.

$f(x) = |x|$... parent function.

- Absolute value function that looks like a “V”.
- Domain $x \in \mathbb{R}$
- Range $f(x) \geq 0$
- Vertex $(0,0)$



Special Note: An absolute value function in vertex form has the equation

$$g(x) = a|x - h| + k$$

- Transformations same as in Part 6.
 - This is $f(x) = |x|$ shifted horizontally to the right by h units, stretched by a factor of a if $a > 1$ or compressed by a factor of a if $a < 1$, and shifted vertically by k units.
- the vertex is at (h, k)
- Domain is $x \in \mathbb{R}$
- $a > 0$, the graph looks like a “V” and has range $f(x) \geq k$
- $a < 0$, the graph looks like a “Λ” and has range $f(x) \leq k$
- The slopes of the lines will always be $\pm a$

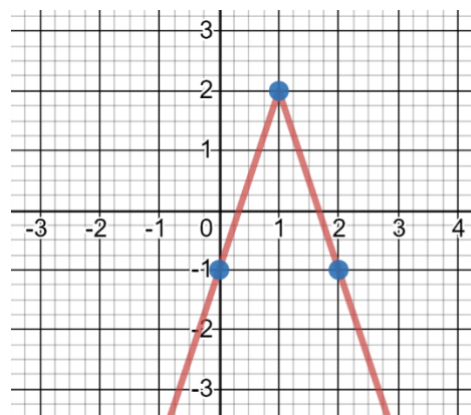
Reminder: Finding the x – intercepts of any function means setting the function equation = 0 and solving for the x – values that make it true.

Example 1: Graph $f(x) = -3|x - 1| + 2$ and find the domain and range.

- 1) Find the vertex $(1, 2)$
- 2) Determine the shape: $-3 \rightarrow \Lambda$ shape
- 3) Create a table and find two more points on either side of the vertex.

Vertex

| x | y |
|-----|-----|
| 0 | -1 |
| 1 | 2 |
| 2 | -1 |



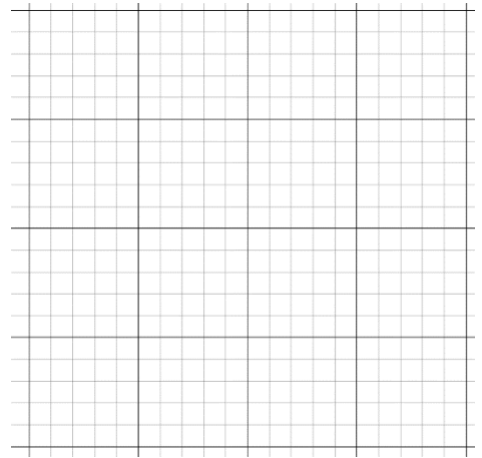
- 4) Graph the 3 points, making sure the slopes are ± 3
- 5) Domain : $x \in \mathbb{R}$
- 6) Range: $y \leq 2$

Example 2: Find the solutions to the equation. $2|x - 3| + 1 = 5$.

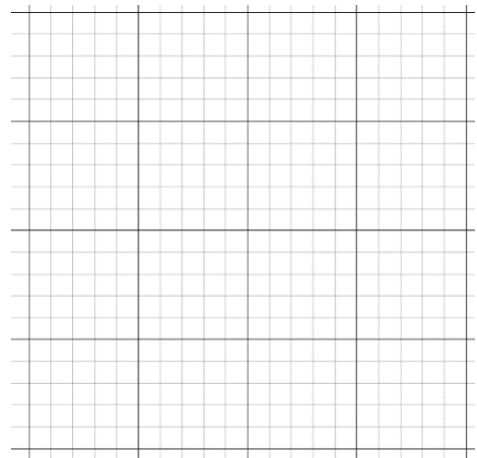
- 1) $2|x - 3| + 1 = 5$ Simplify so the absolute value term equals a constant
 $2|x - 3| = 4$
 $|x - 3| = 2$
- 2) $|x - 3| = 2$ the term inside the absolute value $(x - 3) = \pm$ the constant.
 $x - 3 = \pm(2)$
- 3) $x - 3 = 2$ and $x - 3 = -2$ Simplify
 $x = 5$ and $x = 1$
- 4) The solutions are $x = 1, 5$

FOR YOU TO DO: Graph the following functions and find the domain and range.

1. Graph $f(x) = -\frac{1}{2}|x + 3| - 4$ and find the domain and range.



2. Graph $f(x) = 2|x - 1| + 3$ and find the domain and range.



| | |
|---|---|
| <p>3. Find the solutions to the equation</p> $\frac{1}{3} x-1 +3=2$ | <p>4. Find the x – intercepts of the function</p> $f(x)=2 x-1 -5$ |
| <p>5. Find the solutions to the equation</p> $- x+1 +3=2$ | <p>6. Find the x – intercepts of the function</p> $f(x)=-2 x+3 -8$ |

Part 8: Forms of quadratic functions and graphing.

$f(x) = x^2$ parent function.

- The domain is $x \in \mathbb{R}$.
- The range is $y \geq 0$.
- The vertex is $(0,0)$.

There are two forms of transformed quadratics that we work with.

Vertex form:

$$g(x) = a(x-h)^2 + k$$

- This is the same transformation as reviewed in part 6.
 - This is $f(x) = x^2$ shifted horizontally to the right by h units, stretched by a factor of a if $a > 1$ or compressed by a factor of a if $a < 1$, and shifted vertically by k units.
- Vertex (h,k)
- Domain $x \in \mathbb{R}$
- $a > 0$, the graph looks like a " \cup " and has range $f(x) \geq k$
- $a < 0$, the graph looks like a " \cap " and has range $f(x) \leq k$

Standard form:

$$h(x) = ax^2 + bx + c$$

- Vertex $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$ so $h = \frac{-b}{2a}$ and $k = f\left(\frac{-b}{2a}\right)$
- The " a " values are the same. You can rewrite an equation given in standard form in vertex form.
 - $h(x) = ax^2 + bx + c = a(x-h)^2 + k$
 - $h = \frac{-b}{2a}$
 - $k = f\left(\frac{-b}{2a}\right)$
- Write the equation $f(x) = a(x-h)^2 + k$

Example 1: Rewrite $f(x) = 2x^2 - 8x + 5$ in vertex form.

$$\text{Find } h = -\frac{b}{2a} = -\frac{(-8)}{2(2)} = 2$$

$$\begin{aligned}\text{Find } k &= f(h) = f(-2) \\ &= 2(2)^2 - 8(2) + 5 = -3\end{aligned}$$

$$\text{Write the equation: } f(x) = 2(x-2)^2 - 3$$

Example 2: Graph the function $f(x) = -2x^2 + 4x - 1$ using at least 3 points. Find the domain and range.

Find and graph the vertex and axis of symmetry.

$$h = -\frac{b}{2a} = -\frac{4}{2(-2)} = 1$$

$$k = f(h) = f(1)$$

$$-2(1)^2 + 4(1) - 1 = 1$$

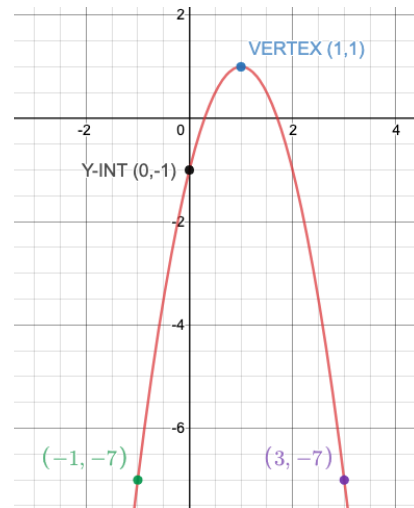
Axis of symmetry is $x = h = 1$

Vertex is **(1,1)**

Find the y -intercept

(0,-1) is the y -intercept

Find at least two other points.



Set up a table, find points on either side of the vertex and graph them.

| | | |
|--------|-----|-----|
| | x | y |
| | -1 | -7 |
| Vertex | 1 | 1 |
| | 3 | -7 |

Confirm the shape is what you expect:

- $a > 0$ means a \cup shape and $a < 0$ means a \cap shape
- The graph is symmetric around the axis of symmetry!
- Graph the equation in Desmos to check your answers.

Domain: $x \in \mathbb{R}$.

Range: $y \leq 3$

FOR YOU TO DO A : Write each quadratic function in vertex form $f(x)$

1. $f(x) = 12x^2 + 5x - 2$

2. $f(x) = x^2 - x - 6$

3. $f(x) = 3x^2 + 14x - 5$

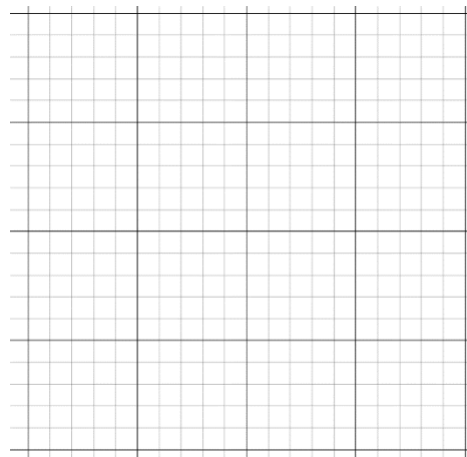
4. $f(x) = 4x^2 - 11x - 3$

5. $f(x) = 5x^2 - 7x - 6$

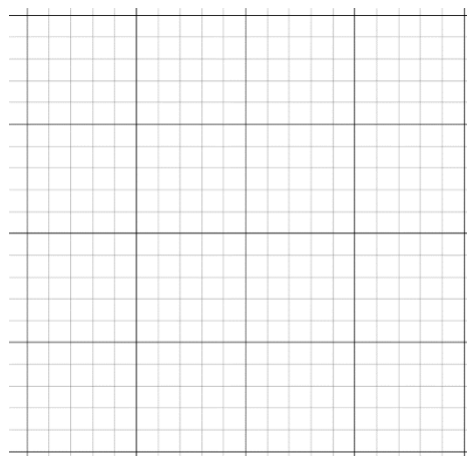
6. $f(x) = 12x^2 + x - 6$

FOR YOU TO DO B: Graph each quadratic function using at least 3 points. Include the vertex, y-intercept and at least two other points. Calculate exact points and if there are fractions, do your best to estimate the values on the graph.

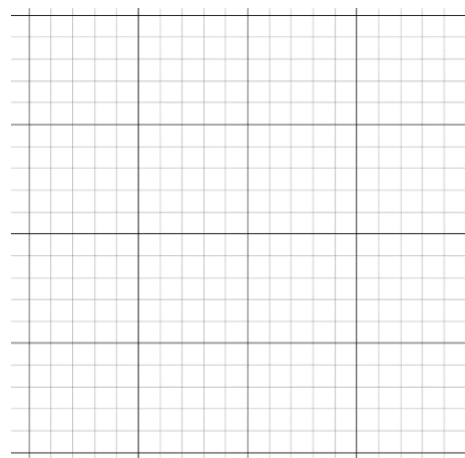
7. $f(x) = 3x^2 + 2$



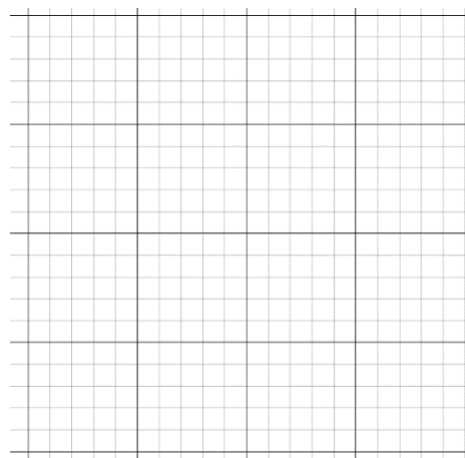
8. $f(x) = x^2 - 2x - 3$



9. $f(x) = \frac{1}{2}(x+2)^2 - 3$



10. $f(x) = 2x^2 - 8x - 11$



Part 9: Factoring quadratics (and polynomials when there's a GCF to factor out first)

Definition: To **factor** means to break a product up into its individual factors, or into the terms multiplied to get to the expression which you are factoring. We factor numbers and polynomials. Sometimes we simply factor out a GCF from an expression. Other times we break down an expression even further. There is also the "Box method" for factoring quadratics.

Example 1: Factoring out a GCF.

Factor $10n^4 - 16n^6$

- The GCF of both terms is $2n^4$
- $10n^4 - 16n^6$ becomes $2n^4(5 - 8n^2)$

Proof: $2n^4(5 - 8n^2)$
 $= 2n^4(5) + 2n^4(-8n^2)$
 $= 10n^4 - 16n^6$

Factor $(x - 2)^2 + (x - 2)^5$

- The GCF of both terms is $(x - 2)^2$
- $(x - 2)^2 + (x - 2)^5$ becomes
 $(x - 2)^2[1 + (x - 2)^3]$

Example 2: Factoring out the GCF and factoring by grouping (there is also the box method)

Factor $6x^2 - 26x - 20$

$= 2(3x^2 - 13x - 10)$

step 1: Factor out 2, the GCF.

step 2: $ac = (3)(-10) = -30$

factors of -30 whose sum is -13 are:

$-15 + 2 = -13.$

$= 2(3x^2 - 15x + 2x - 10)$

step 3: replace $-13x$ with $-15x + 2x$

$= 2[(3x^2 - 15x) + (2x - 10)]$

step 4: group

$= 2[3x(x - 5) + 2(x - 5)]$

step 5: factor the GFC from each group

$= 2(3x + 2)(x - 5)$

step 6: factor the common linear term from each group and
write factored form.

Notice that when you factor out the **(x-5)** from the individual groups, what remains in the square brackets is $[3x+2]$... the "new" factor. The linear term in each group must be the same for grouping to work!

FOR YOU TO DO: Factor each quadratic completely.

1. $x^2 - 9$

2. $x^2 - x - 6$

3. $5x^3 + 15x^2 - 20x$

4. $4x^2 - 11x - 3$

5. $3x^2 + 14x - 5$

6. $5x^2 - 7x - 6$

| | |
|---------------------|----------------------|
| | |
| 7. $9x^2 - 18x + 8$ | 8. $12x^2 + x - 6$ |
| 9. $12x^2 + 5x - 2$ | 10. $8x^2 - 22x + 5$ |

Part 10: Solving quadratics.

Solving any equation means setting the equation = 0 and finding the x – values that make it true.

Finding the x – intercepts of a function means setting the function equation = 0 ($y = f(x) = 0$) and finding the x – values that make it true.

There are 4 techniques for solving quadratics:

- 1) Graphing (a great place to start for checking your work)
- 2) Factoring (grouping or box method)
- 3) Completing the square, and
- 4) The quadratic formula.

Examples:

- 1) Solve $2x^2 + 5x = 3$ by graphing.

- a. Write the equation as a function=0 and find the x = intercepts using Desmos or the Ti_Nspire.

i. $2x^2 + 5x - 3 = 0$

- 2) Solve $2x^2 + 5x = 3$ by factoring.

a. $2x^2 + 5x - 3 = 0$

Write the equation as a function=0 and factor using grouping or the box method.

b. $(2x - 1)(x + 3) = 0$

Factor using grouping or the box method

- c. Find the values of x that make each factor = 0.

i. $(x + 3) = 0 \rightarrow x = -3$

ii. $(2x - 1) = 0 \rightarrow x = \frac{1}{2}$

- 3) Solve $2x^2 + 4x = 3$ by completing the square

a. $2x^2 + 4x = 3$.

Put all the x terms on one side and the constant term on the other.

b. $x^2 + 2x = \frac{3}{2}$

Make the x^2 coefficient = 1.

c. $x^2 + 2x + 1 = \frac{3}{2} + 1$ Add $\left(\frac{b}{2}\right)^2$ to both sides.

d. $(x + 1)^2 = \frac{5}{2}$

Factor the left side as a perfect square $\left(x + \frac{b}{2}\right)$; simplify the constants

e. $x + 1 = \pm \sqrt{\frac{5}{2}} = \pm \sqrt{\frac{10}{4}} = \pm \frac{\sqrt{10}}{2}$ Take the square root of both sides

f. $x = -1 \pm \frac{\sqrt{10}}{2}$ Solve for x .

4) Solve $2x^2 + 4x = 3$ using the quadratic formula.

a. $2x^2 + 4x - 3 = 0$

Rewrite equation $= 0$

b. $x = \frac{-4 \pm \sqrt{4^2 - 4(2)(-3)}}{2(2)}$

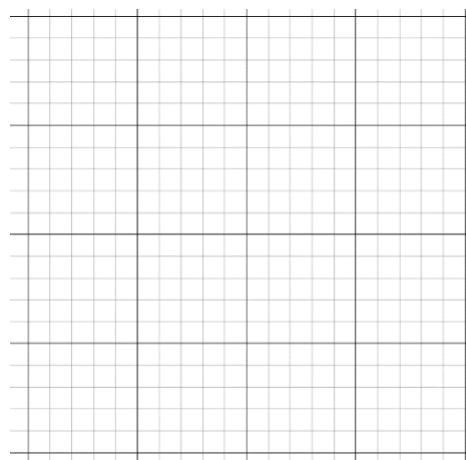
Apply quadratic formula $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

c. $x = \frac{-4 \pm \sqrt{40}}{4} = \frac{-4 \pm 2\sqrt{10}}{4} = -1 \pm \frac{\sqrt{10}}{2}$ Simplify.

FOR YOU TO DO: Solve each quadratic equation using the methods specified.

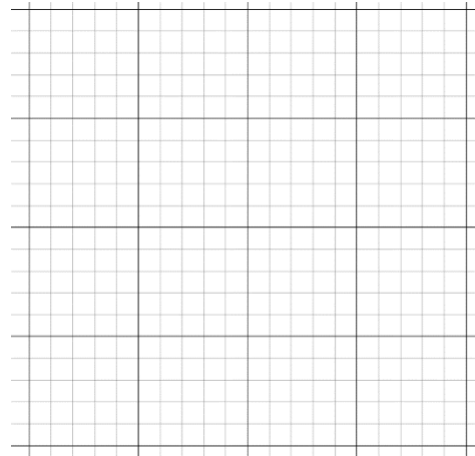
- You will always get an answer using completing the square and the quadratic formula that are the same.
- You may not be able to factor. That's ok. Think about why!
- Graphing may only give you an estimate of your solutions. This happens when your answer using completing the square or the quadratic formula has a radical term.
- Check that your answers/estimates are almost exactly the same!

1. $x^2 - x - 6 = 0$ Use at least one of the following: factoring, completing the square, or the quadratic formula. Check your answer by graphing.



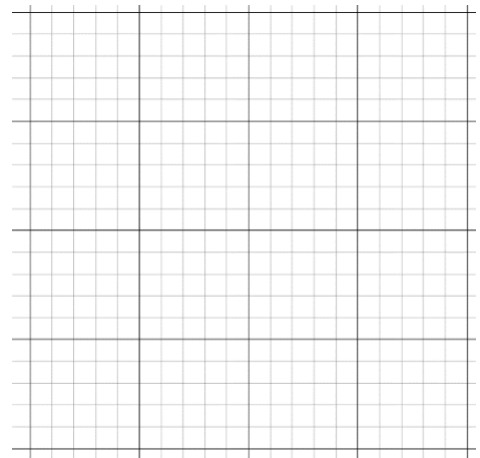
Sketch your graphing results

2. $4x^2 - 11x - 3 = 0$. Use at least one of the following: factoring, completing the square, or the quadratic formula. Check your answer by graphing.



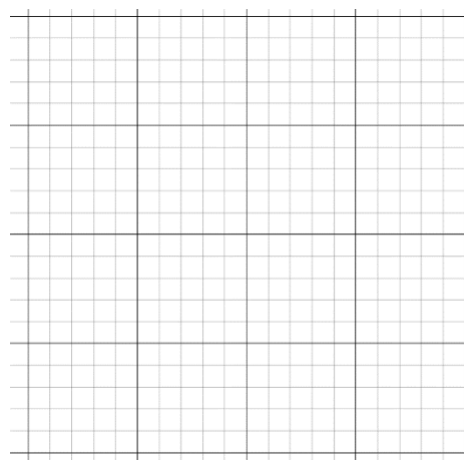
Sketch your graphing results

3. $5x^3 + 15x^2 - 20x = 0$ Use at least one of the following: factoring, completing the square, or the quadratic formula. Check your answer by graphing.



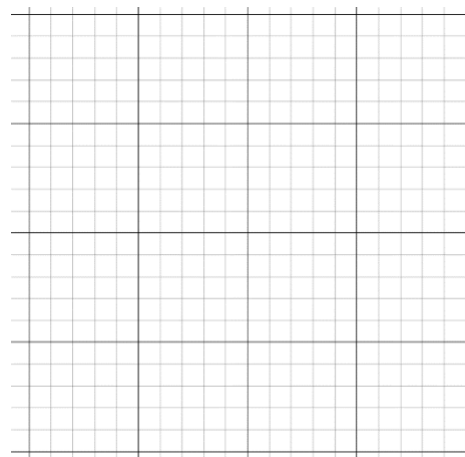
Sketch your graphing results

4. $12x^2 + x - 6 = 0$. Use at least one of the following: factoring, completing the square, or the quadratic formula. Check your answer by graphing.



Sketch your graphing results

5. $3x^2 + 14x - 5 = 0$. Use at least one of the following: factoring, completing the square, or the quadratic formula. Check your answer by graphing.



Sketch your graphing results

Part 11: Factoring Special Cases

When we factor, sometimes we see patterns and follow rules to make our task easier. Such patterns, or identities, are usually memorized and applied when they're recognized. Practicing them repeatedly helps us see them better when they pop up among other problems.

In Algebra 2A you learned the difference of squares and perfect square trinomials. (Do problems 2, 4, 7, 9)

In Algebra 2B you learned the sum and difference of cubes and perfect cubes. (Do problems 1, 3, 5, 6, 8, 10)

| Difference of Squares | Sum and Difference of Cubes |
|--|--|
| $a^2 - b^2 = (a + b)(a - b)$ | $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ |
| Perfect Square Trinomials | Perfect Cubes |
| $(a + b)^2 = a^2 + 2ab + b^2$ $(a - b)^2 = a^2 - 2ab + b^2$ | $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$ |

FOR YOU TO DO:

Factor each of the following expressions completely.

| | |
|--------------------------|------------------|
| 1) $x^3 + 64$ | 2) $81x^2 - 100$ |
| 3) $x^3 + 3x^2 + 3x + 1$ | 4) $x^4 - 49y^2$ |

| | |
|-----------------------|--|
| 5) $x^6 - 276$ | 6) $6x^5 - 36x^3$ |
| 7) $x^8 - 256y^4$ | 8) $x^3 - 6x^2 + 12x - 8$ |
| 9) $(2x + 5)^2 - 121$ | 10) $125x^3 + (x - 6)^3$ This is a challenge problem! |

Part 12 : Polynomials, terminology, end behaviors

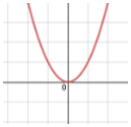


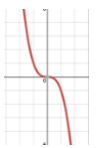
A polynomial function has exponents that are **whole numbers**, and coefficients that are **real numbers**. $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x^1 + a_0 (x^0)$ don't forget, $x^0 = 1$

The **leading coefficient** in this example is a_n

The **constant term** is a_0

A polynomial is in **standard form** if its terms are written in order of exponents from highest to lowest (left to right)

End behavior (EB)

| | | | |
|--|---|---|---|
| Leading coefficient positive Even degreed function EB: up, up |  | Leading coefficient positive Odd degreed function EB: down, up |  |
| Leading coefficient negative Even degreed function EB: down, down |  | Leading coefficient negative Odd degreed function EB: up, down |  |

FOR YOU TO DO:

| | |
|--|---|
| 1. Is this a polynomial? $f(x) = x^7 - 3x^{10} + \sqrt{2} \cdot x^3$ | 2. What degree is this function? $f(x) = 32x - 231x^5 + x^2$ |
| 3. What is the leading coefficient of this function? $f(x) = 6x^2 - 10x^3 + 2$ | 4. What are the end behaviors of this functions? $f(x) = 16x^4 - x^3 + 2x^2$ |
| 5. What are the end behaviors of this functions? $f(x) = 5x^2 - x^7 + 2x^4 - 9$ | 6. Put this into standard form $f(x) = 3x - 14x^5 + 2x^3 + 5 - 8x^2$ |

Part 13 : Radicals

If $a^n = b$, with a and b real numbers and n is a positive integer, then a is an **nth root** of b

If n is odd, there is only one real root of b

If n is even and b is positive, there are two real n th roots of b .

If n is even and b is negative there are no real roots, only imaginary.

In general, $\sqrt[n]{x^m} = x^{\frac{m}{n}}$ for any positive integer n .

Properties of exponents apply

Examples: solve

$$27^{\frac{1}{3}}$$

Answer: 3

Evidence: $\sqrt[3]{27} = 3$

Write in radical form

$$x^{\frac{2}{7}}$$

Answer: $\sqrt[7]{x^2}$

FOR YOU TO DO:

| | |
|--|---|
| 1. Simplify $(-32)^{\frac{4}{5}}$ | 2. Write in exponential form $\sqrt[3]{x^7}$ |
| 3. Write in radical form $x^{\frac{5}{9}}$ | 4. Simplify $4^{1.5}$ |
| 5. Write in radical form $(x^3)^{\frac{1}{5}}$ | 6. Simplify $(-8x\sqrt{xy})^{\frac{2}{3}}$ |

Part 14 : Adding and Subtracting Rational Expressions, stating restrictions

When adding and subtracting rational expressions

1. Factor denominators to help find LCD
2. Restrictions are values that would make the denominator zero (discontinuities)
3. Expand the numerators and put together
4. Eliminate any common factors

Examples:

$$\frac{x}{x-1} + \frac{2x-1}{x^2-3x+2} = ?$$

Answer: $\frac{x+1}{x-2}$ $x \neq 1$ and $x \neq 2$

Evidence:
$$\frac{x}{(x-1)} \left(\frac{(x-2)}{(x-2)} \right) + \frac{2x-1}{(x-2)(x-1)}$$

$$\frac{x^2-1}{(x-1)(x-2)} = \frac{(x-1)(x+1)}{(x-1)(x-2)}$$

$$\frac{x+2}{x^2-2x} - \frac{x+2}{2x-4} = ?$$

Answer: $\frac{-x-2}{2x}$ $x \neq 2$ and $x \neq 0$

Evidence:
$$\frac{x+2}{x(x-2)} \left(\frac{2}{2} \right) - \frac{x+2}{2(x-2)} \left(\frac{x}{x} \right)$$

$$\frac{2x+4-(x^2+2x)}{2x(x-2)}$$

$$\frac{-(x^2-4)}{2x(x-2)} = \frac{-(x-2)(x+2)}{2x(x-2)}$$

$$= \frac{-x-2}{2x(x-2)}$$

FOR YOU TO DO:

Add or subtract the rational expression

1. $\frac{5}{6x^2} + \frac{x}{4x^2 - 12x}$

2. $\frac{x+1}{x^2+4x+4} - \frac{2}{x^2-4}$

3. $\frac{10x}{3x^2-3} + \frac{4}{x-1} + \frac{5}{6x}$

4. $\frac{2x+1}{x^2+8x+16} - \frac{3}{x^2-16}$

Part 15: General Rational Functions and their domain and range

The analysis:

- Find the horizontal asymptote(HA) (end behaviors):
 If the degree of the numerator < degree of the denominator HA: $y=0$
 If the degree of the numerator = degree of the denominator HA: $y = \frac{\text{leading coefficient of numerator}}{\text{leading coefficient of denominator}}$
 If the degree of the numerator > degree of the denominator HA: none (oblique or slant asymptote)
- Find the y-intercept (set $x=0$, solve for y)
- Factor the numerator and denominator
- x-intercepts are the zeros of the numerator (non-removable)
- Zeros of the denominator are either
 - Vertical asymptotes (VA's) – nonremovable (discontinuities)
 - Holes - removable discontinuities (these values are not vertical asymptotes)
- Domain: all real numbers except zeros of the original denominator

Example 1:

$$y = \frac{3x^2}{x^2 - 4}$$

Horizontal asymptote: $y = \frac{3}{1} = 3$

y-intercept: $y = \frac{3(0)}{0 - 4} = \frac{0}{-4} = 0$

$$y = \frac{3x^2}{(x - 2)(x + 2)}$$

x-intercept: $(0,0)$

vertical asymptotes: $x=2$, $x=-2$

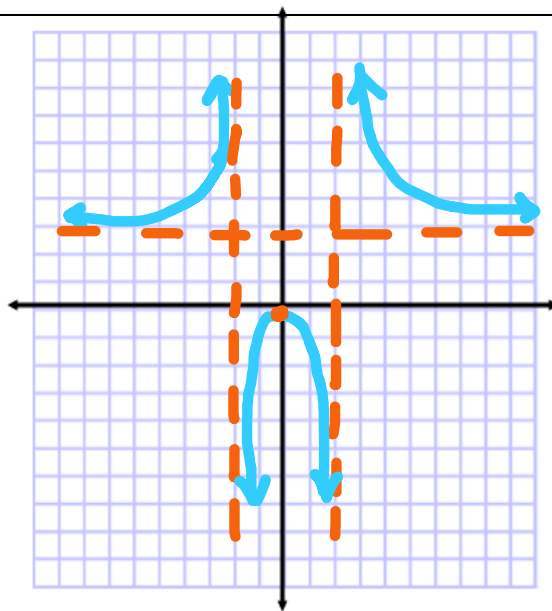
holes: none

domain: $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

put all of this information on the graph

connect the dots, approach the asymptotes, where you're not sure, use sign analysis or graph a point.

Don't forget: you can't create new x-intercepts, you can't cross vertical asymptotes, but you can cross horizontal asymptotes (they are really end behaviors)



Example 2:

$$y = \frac{3x^2 - 5x - 2}{x^2 + 3x - 10}$$

Horizontal asymptote: $y = \frac{3}{1} = 3$

y-intercept:

$$y = \frac{3(0) - 5(0) - 2}{0 + 3(0) - 10} = \frac{-2}{-10} = \frac{1}{5}$$

$$\left(0, \frac{1}{5}\right)$$

Factor: $y = \frac{(3x+1)(x-2)}{(x-2)(x+5)}$

x-intercept $\left(-\frac{1}{3}, 0\right)$

vertical asymptotes: $x = -5$

holes: $(2, 1)$

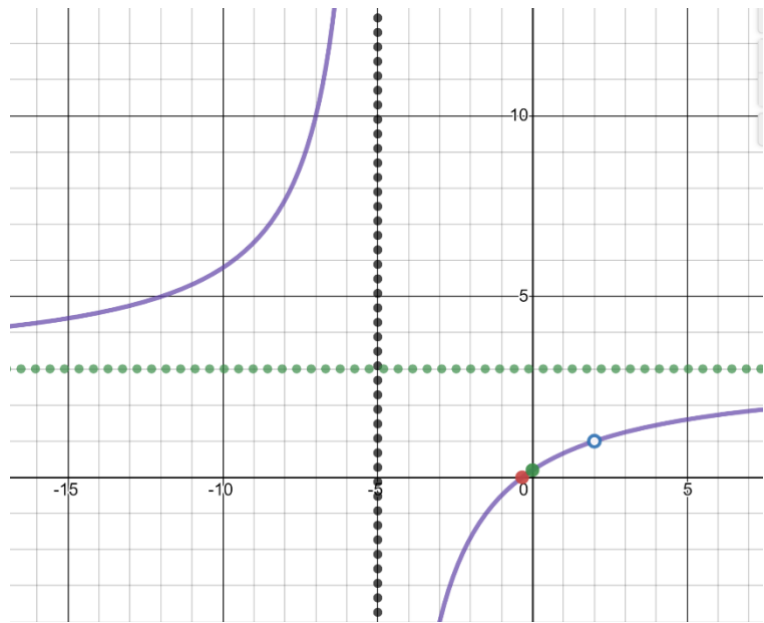
domain: $(-\infty, -5) \cup (-5, 2) \cup (2, \infty)$

put all of this information on the graph

connect the dots, approach the asymptotes, where you're not sure, use sign analysis or graph a point.

Don't forget: you can't create new x-intercepts, you can't cross vertical asymptotes, but you can cross horizontal asymptotes (they are really end behaviors).

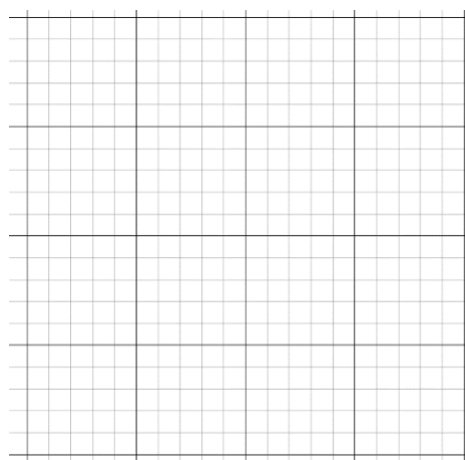
Don't forget: since domain includes everything except -5 and 2, the function exists when $x < -5$, you need to decide where the graph is, above or below the horizontal asymptote (can't be below because it would create an x-intercept that doesn't exist - or try out a point below -5)



FOR YOU TO DO: Graph the following rational functions.

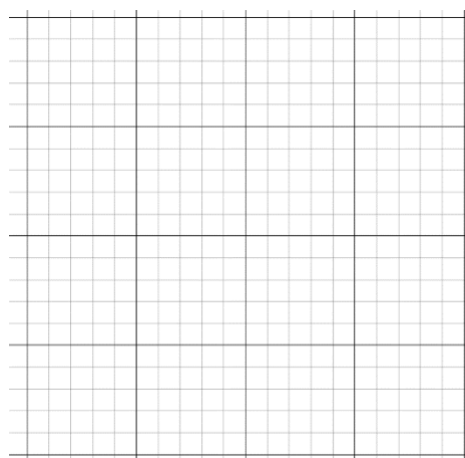
List: horizontal asymptote, vertical asymptote(s), x-int(s), y-int, hole(s), domain (write none if it doesn't exist)

1. $y = -\frac{4}{x^2 - 3x}$



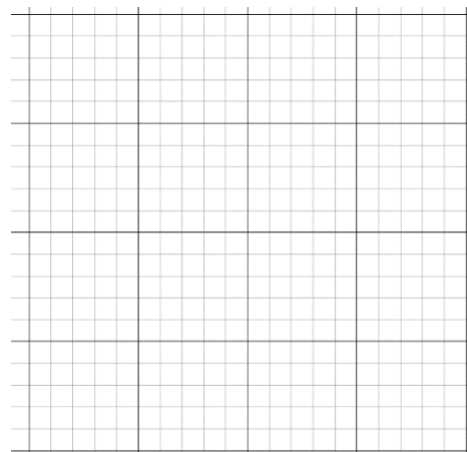
Sketch your graphing results

2. $y = \frac{2x^2 + 10x + 12}{x^2 + 3x + 2}$



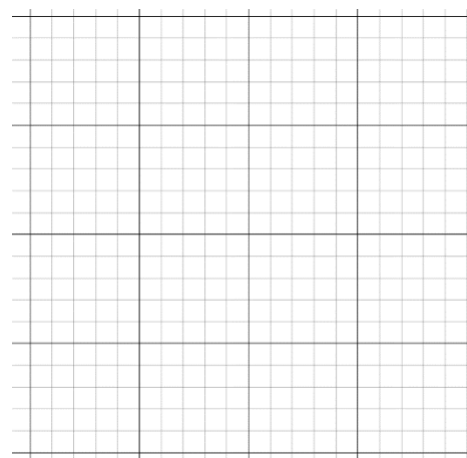
Sketch your graphing results

3. $y = \frac{4x}{x^2 - 5x - 24}$



Sketch your graphing results

4. $y = \frac{3x^2 - 12x}{x^2 - 2x - 3}$



Sketch your graphing results

Part 16 : Exponential functions

An exponential function has the general form $y = ab^x$, $a > 0$, $b > 0$ and $b \neq 1$

When $b > 1$, $f(x)$ is a growth function

When $0 < b < 1$, $f(x)$ is a decay function

For either case, for the parent function: the y-intercept is $(0, a)$, the domain is all real numbers and the asymptote is $y = 0$, and the range is $y > 0$

Translation of exponential functions:

$$y = ab^{(x-h)} + k$$

Examples: graph: $y = -2 \cdot \left(\frac{1}{2}\right)^{x-3} + 1$

Parent function: $y = \left(\frac{1}{2}\right)^x$

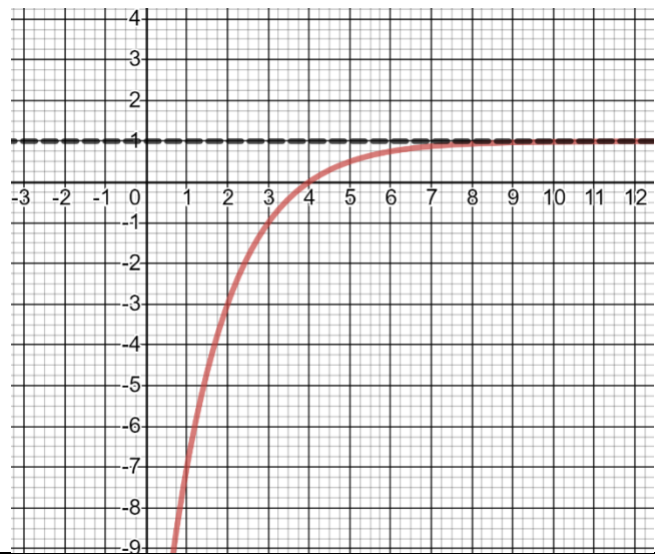
Domain: all real numbers, $(-\infty, \infty)$

Range: $(-\infty, 1)$

x-int: $(4, 0)$

y-int: $(0, -15)$

asymptote: $y = 1$



FOR YOU TO DO

1. Graph $y = 3 \cdot 2^{x+2} - 5$

Parent function:

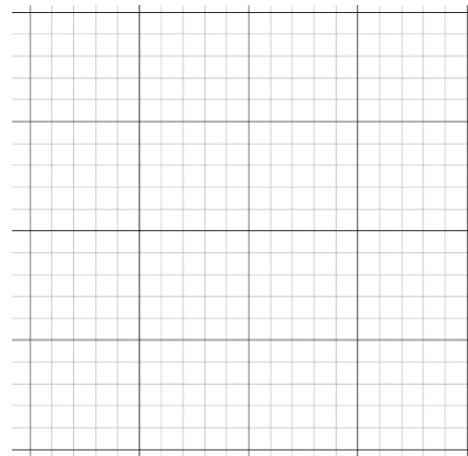
Domain:

Range:

$x = \text{int}(s)$:

$y = \text{int}$:

Asymptote(s):



Sketch your graphing results

Part 17 : Logarithmic Functions

Defining the logarithm:

A logarithm base b of a positive of a positive number x satisfies the following definition

For $b > 0, b \neq 1, \log_b x = y$ if and only if $b^y = x$

domain of the function $(0, \infty)$

Range is all real numbers

Vertical Asymptote is $x = 0$

Transformations: $y = a \log_b(x - h) + k$

Properties of logarithms

Product Property : $\log_b mn = \log_b m + \log_b n$

Quotient Property: $\log_b \frac{m}{n} = \log_b m - \log_b n$

Power Property: $\log_b m^n = n \log_b m$

To solve, logarithmic/exponential equations there are many different approaches, here are some:

1. Match bases (then the exponents would be equal) $2^x = 2^7, x = 7$
2. Use definition of a log to solve: $\log_2 8 = x \rightarrow 2^x = 8 \rightarrow 2^x = 2^3 \rightarrow x = 3$

$$3^x = 21 \rightarrow \log 3^x = \log 21 \rightarrow x \log 3 = \log 21$$

3. Use properties of logs: $x = \frac{\log 21}{\log 3} (\text{calculator})$

Examples:

$$4 \log_3 x = 28$$

$$\text{Answer: } x = 3^7$$

Evidence: $4 \log_3 x = 28$ isolate the logarithm
 $\log_3 x = 7$
 $3^7 = x$

$$\ln x + \ln(x - 2) = 1$$

$$\text{Answer: } 1 + \sqrt{1 + e} \approx 2.928$$

Evidence:

$$\begin{aligned} \ln x + \ln(x - 2) &= 1 \\ \ln(x^2 - 2x) &= 1 \text{ product rule} \\ e^1 &= x^2 - 2x \\ x^2 - 2x - e &= 0 \end{aligned}$$

Solve using the quadratic formula

Example: Graph: $f(x) = \log_3(x - 2) - 1$

Parent function $f(x) = \log_3 x$
 $3^y = x$

Shifted 2 to the right, and down 1

So (1,0) transforms to (3,-1)

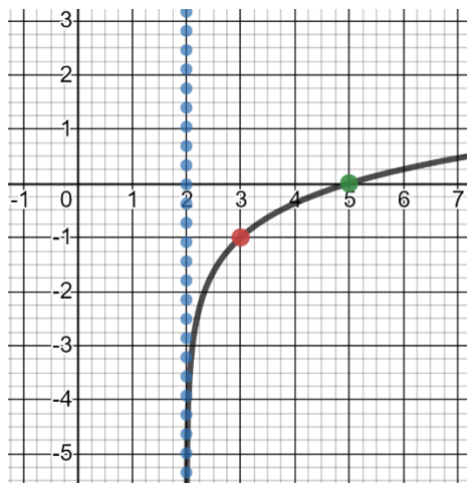
Domain: $(2, \infty)$

Range: $(-\infty, \infty)$

y-int: none

x-int: (0,5)

vertical asymptote: $x=2$



FOR YOU TO DO:

Solve for x

| | |
|-------------------------------------|--|
| 4. $4^{3x} = 8^{x+1}$ | 5. $2^x = 7$ |
| 6. $\log_3(5x - 1) = \log_3(x + 7)$ | 7. $\log_5(3x + 1) = 2$ |
| 8. $\log 5x + \log(x - 1) = 2$ | 9. $0.25^x - 0.5 = 2$ |
| 10. $4 - 2e^x = -23$ | 11. $\ln(x + 5) = \ln(x - 1) - \ln(x + 1)$ |

12. Graph: $y = 2\log_2(x - 1) + 3$

Parent function:

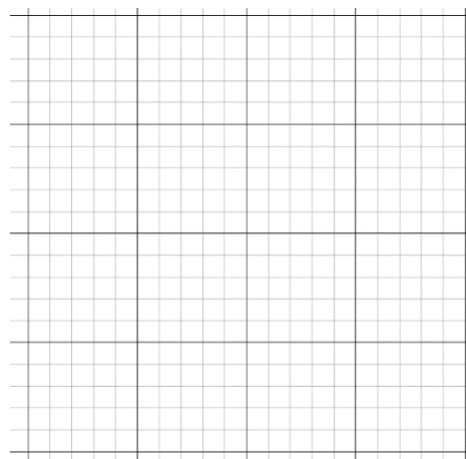
Domain:

Range:

x int(s):

y int:

asymptote(s):



Sketch your graphing results

Part 18: Right Triangle Trigonometry (Do not do if you have not had geometry)

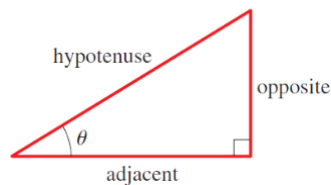
When we need to find missing angle measures or missing side lengths in right triangles, we use SOHCAHTOA or, if possible, the Special Right Triangle rules.

SOHCAHTOA. For any acute angle θ ...

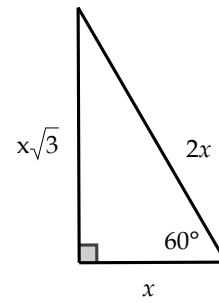
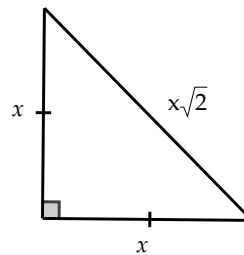
$$\text{SOH: } \sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\text{CAH: } \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

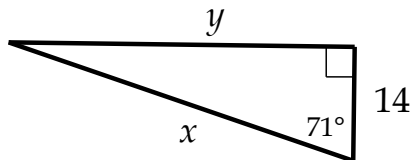
$$\text{TOA: } \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$



Special Right Triangles. We have two “special” right triangles... 45-45-90 (Isosceles Right Triangles) and 30-60-90 triangles. These have special relationships among their side lengths.



Example 1: Solve for x and y in the triangle below.



$$\tan 71 = \frac{y}{14}$$

$$14 \tan(71) = y$$

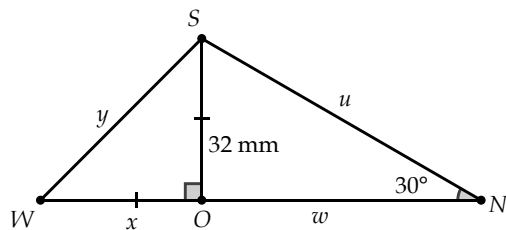
$$40.65 \approx y$$

$$\cos 71 = \frac{14}{x}$$

$$x = \frac{14}{\cos 71}$$

$$x \approx 43$$

Example 2: Solve for the variables in the figure below



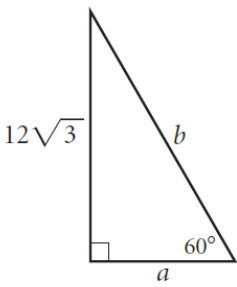
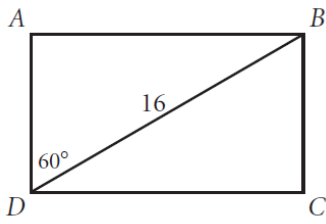
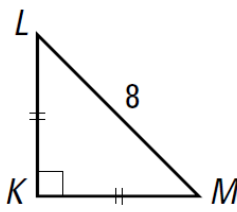
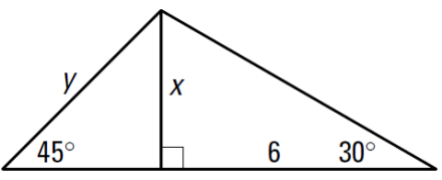
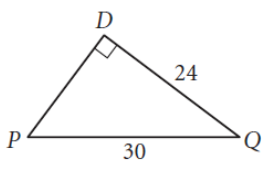
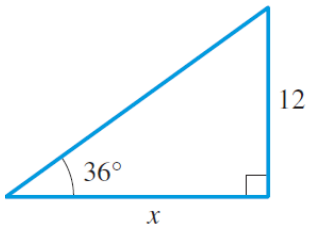
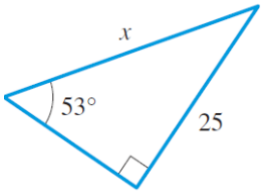
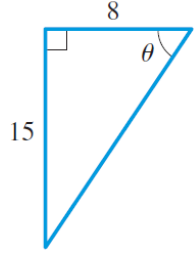
$x = 32 \text{ mm}$ as it is equivalent to SO

$y = 32\sqrt{2} \text{ mm}$ as it is $\sqrt{2}$ times the leg

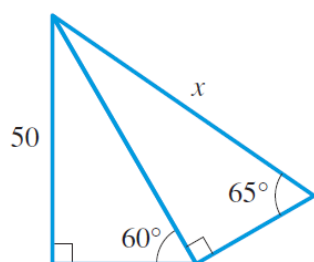
$u = 64 \text{ mm}$ as it is double the length of the shorter leg

$w = 32\sqrt{3} \text{ mm}$ as it is $\sqrt{3}$ times the shorter leg

FOR YOU TO DO (all answers including radicals should be left in rationalized radical form):

| | |
|---|---|
| <p>1.</p> <p>$a = \underline{\hspace{2cm}}, b = \underline{\hspace{2cm}}$</p>  | <p>2. Determine the dimensions of the rectangle.</p> <p>AB = CD = $\underline{\hspace{2cm}}$ AD = BC = $\underline{\hspace{2cm}}$</p>  |
| <p>3. Determine the lengths.</p> <p>KL = $\underline{\hspace{2cm}}$ KM = $\underline{\hspace{2cm}}$</p>  | <p>4. Use the triangle below.</p> <p>$x = \underline{\hspace{2cm}}$ $y = \underline{\hspace{2cm}}$</p>  |
| <p>5.</p> <p>$\sin Q = \underline{\hspace{2cm}}$</p> <p>$\cos Q = \underline{\hspace{2cm}}$</p> <p>$\tan Q = \underline{\hspace{2cm}}$</p>  | <p>6. Calculate the value of x to the nearest tenth.</p> <p>$x \approx \underline{\hspace{2cm}}$</p>  |
| <p>7. Calculate the value of x to the nearest tenth.</p> <p>$x \approx \underline{\hspace{2cm}}$</p>  | <p>8. Use inverse trig functions to calculate the value of θ to the nearest degree.</p> <p>$\theta \approx \underline{\hspace{2cm}}$</p>  |

9. Calculate the value of x to the nearest tenth. $x \approx$ _____



10. Calculate the value of x to the nearest tenth. $x \approx$ _____

